 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **STATISTICS**

SECOND SEMESTER – APRIL 2012

# ST 2812 - TESTING STATISTICAL HYPOTHESES

Date : 19-04-2012 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

SECTION - A

Answer ALL questions. Each carries TWO marks: (10 x 2 = 20 marks)

1. How do the Loss and Risk functions quantify the consequences of decisions?

2. Specify the three elements required for solving a decision problem.

3. Describe a situation where the decision rule remains invariant or symmetric.

4. Define Bayes Rule and Bayes Risk.

5. Illustrate that the consequences of Type I error and Type II error are quite different.

6. Define Most Powerful Test of level α.

7. Write UMPT for one parameter exponential family for testing

(i) H: θ ≤ θ0 versus K: θ > θ0 when Q (θ) is increasing

(ii) H: θ ≥ θ0 versus K: θ < θ0 when Q(θ) is decreasing.

8. When do we say that a test  has Neyman Structure?

9. State any two asymptotic results regarding likelihood equation solution.

10. What is an invariant test?

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks: (5 x 8 = 40 marks)

11. Distinguish between randomized and non-randomized tests and give an example for

each test.

12. Let ‘N’ be the size of a lot containing ‘D’ defectives, where ‘D’ is unknown. Suppose a

sample of size ‘n’ is drawn and the number of defectives ‘X’ in the sample is observed.

Obtain UMPT of level α for testing H: D ≤ D0 versus K: D > D0.

13. Let ‘X’ denote the number of events observed during a time interval of length ‘τ’ in a

Poisson process with rate ‘λ’. When τ = 1, at 5% level, find the power at λ = 1.5 of the

UMPT for testing H: λ ≤ 0.5 versus K: λ > 0.5.

14. Obtain the UMPUT for H: p = p0 versus K: p ≠ p0 in the case Binomial distribution

with known ‘n’ and deduce the ‘side conditions’ that are required to be satisfied.

15. State and prove a necessary and sufficient condition for similar tests to have Neyman

structure.

16. If X ~ P (λ1) and Y ~ P (λ2) and are independent, then compare the two Poisson populations

through UMPUT for H: λ1 ≤ λ2 versus K: λ1 > λ2, by taking random sample from P (λ1) and

P (λ2) of sizes ‘m’ and ‘n’ respectively.

17. Show that a test is invariant if and only if it is a function of a maximal invariant statistic.

18. Using a random sample of size ‘n’ from N(μ, 1), derive the likelihood ratio test of level α

for testing H: μ = 0 against K: μ ≠ 0.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks: ( 2 x 20 = 40 marks)

19. State and prove the existence, necessary and sufficiency parts of Neyman-Pearson

Fundamental Lemma.

20(a) For a two decision problem, with zero loss for a correct decision, prove that every

minimax procedure is unbiased. (10)

(b) Prove that an unbiased procedure is minimax if Pθ(A) is a continuous function of θ

for every event ‘A’ and there is a common boundary point of Θ0 and Θ1. (10)

21. Let X1, … , Xn be a random sample from E(a, b), where ‘a’ is unknown and ‘b’ is

known. Using the UMPT for testing H: θ = θ0 versus K: θ ≠ θ0 in U(0, θ), obtain the

UMPT for testing H: a = a0 versus K: a ≠ a0 and find its power function.

22(a) Derive the conditional UMPUT of level α for testing the independence of attributes

in a 2 x 2 contingency table. (16)

(b) Discuss the criteria for choosing the value of significance level α. (4)

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